Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Problem Solving-I

Subject Code: 5SC02PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 06/05/2017** **Time: 02:00 To 05:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a. Solve: $y + p^2 = xp$ **(02)**

b. Transform $x^2 y'' - 2xy' + 2y = 0$ into a self adjoint equation. **(02)**

c. Suppose T be the linear operator on R^3 defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$ then T is invertible. **(02)**

d. The function $f(z) = \bar{z}$ is not analytic at any point. – True or False? **(01)**

Q-2 Attempt all questions **(14)**

a. For the Strum-Lioville problem $X'' + \lambda X = 0, X(0) = 0, X(\pi) = 0$ obtain the eigenfunctions and the corresponding eigenvalues. **(07)**

b. Find modulus and argument of $z = \dfrac{\left(3 - \sqrt{2}\,i\right)^2}{1 + 2i}$. **(04)**

c. Find the value class of the quadratic form $6x_1^2 + 3x^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$. **(03)**

OR

Q-2 Attempt all questions **(14)**

a. Solve: i) $(2x\log x - xy)dy + 2ydx = 0$ **(04)**

ii) $(D^2 - 4D + 4)y = xe^{2x}$ **(03)**

b. Find the characteristic and minimal polynomial of $A=\begin{bmatrix}4 & -2 & 2\\ 6 & -3 & 4\\ 3 & -2 & 3\end{bmatrix}$. **(04)**

c. Separate $\tan^{-1}(x+iy)$ into real and imaginary parts. **(03)**

Q-3 Attempt all questions **(14)**

a. If $\frac{dy}{dx}+2y\tan x=\sin x$ and $y\left(\frac{\pi}{3}\right)=0$ then find the maximum value of $y(x)$. **(04)**

b. If S is defined by the rectangle $|x|\le a, |y|\le b$ then show that the function $f(x,y)=x\sin y+y\cos x$ satisfy the Lipschitz condition. Find the Lipschitz constant. **(03)**

c. Let *V(R)* be the vector space of all complex no $a+ib$ over the field of R and let T be a mapping from *V(R)* to $V_2(R)$ defined by $T(a+ib)=(a,b)$ then prove that T is isomorphism. **(04)**

d. Prove that $\tan^{-1} z=\frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$ **(03)**

OR

Q-3 Attempt all questions **(14)**

a. Solve: $\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}$ **(04)**

b. Show that the function $y(x)=cx^2+x+3$ is not an unique solution of the initial value problem $x^2y''-2xy'+2y=6$ with $y(0)=3, y'(0)=1$ on $(-\infty,\infty)$. **(03)**

c. Find inverse of matrix by Gauss Jordan method, where $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & 2 & 1\end{bmatrix}$. **(04)**

d. If $u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ then prove that $\tanh\left(\frac{u}{2}\right)=\tan\frac{\theta}{2}$. **(03)**

SECTION - II

Q-4 Answer the Following questions: **(07)**

a. If $T:P_2(R)\to P_3(R)$ defined by $T(f(x))=2f'(x)+3\int_0^x f(t)\,dt$ then find rank of *T*. **(02)**

b. Prove that the system of three vectors $(1,3,2),(1,-7,-8)\&(2,1,-1)$ of $V_3(R)$ is linearly dependent. **(02)**

c. The value of the integral $\int_C \frac{dz}{z(z-2)},\ C:|z|=1$ is ________. **(02)**

d. Find the real part of $(\sin x + i\cos x)^7$. **(01)**

Q-5 Attempt all questions **(14)**

a. Find A^{-1} by using Cayley-Hamilton theorem, where $A=\begin{bmatrix}4 & 3 & 1\\ 2 & 1 & -2\\ 1 & 2 & 1\end{bmatrix}$. **(05)**

b. Using Cauchy's theorem, evaluate $\oint_C \frac{2z-3}{z^3-3z^2+4}dz$, where C is the circle **(05)**

i) $|z|=\frac{3}{2}$ and ii) $|z-3|=2$.

c. Find Taylor's expansion of $f(z)=\frac{2z^3+1}{z^2+z}$ about the point $z=i$. **(04)**

OR

Q-5 Attempt all questions **(14)**

a. Write the Jordan canonical form of $\begin{bmatrix}7 & 4 & -1\\ 4 & 7 & -1\\ -4 & -4 & -4\end{bmatrix}$. **(05)**

b. Using Cauchy's integral formula, evaluate $\oint_C \frac{z^4}{(z+1)(z-i)^2}dz$, where C is the ellipse $9x^2+4y^2=36$. **(05)**

c. Check whether the following functions are analytic or not. **(04)**

i) $f(z)=\frac{z}{\bar{z}}$ ii) $f(z)=\sin z$

Q-6 Attempt all questions **(14)**

a. Solve $y''+4y=\tan 2x$ by using the method of variation of parameters. **(05)**

b. Find eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. **(05)**

c. Find Laurent's expansion of $f(z)=\frac{7z-2}{(z+1)z(z-2)}$ in the annulus $1<z+1<3$ **(04)**

OR

Q-6 Attempt all Questions **(14)**

a. Solve the simultaneous equations $\frac{dy}{dt}+2y+\sin t=0,\ \frac{dy}{dt}-2x-\cos t=0$; where $x(0)=0, y(0)=1$ **(05)**

b. Solve the following system of linear equation $x+y+z=3,\ x+2y+3z=4, x+4y+9z=6$. **(05)**

c. Find the bilinear transformation that maps the points $0,1,i$ in z-plane onto the points $1+i,-i,2-i$ in the w-plane. **(04)**